In: Proceedings of International Conference on Artificial Neural Networks ICANN 1995 vol2. Paris

ANALYSIS OF THE INFLUENCE OF ADAPTATIVE TIME CONSTANTS ON THE DYNAMICAL BEHAVIOUR OF RECURRENT NEURAL NETWORKS

Jean-Philippe Draye*,[1], Davor Pavisic*, Gaëtan Libert*, Guy Cheron°

* Faculté Polytechnique de Mons
"Parallel Information Processing" Laboratory
Rue de Houdain, 9 - B7000 MONS
BELGIUM

° Université Libre de Bruxelles
Laboratory of Biomechanics ISEPK
Avenue Paul Heger, 28 - B1050 BRUSSELS
BELGIUM

Abstract : *We explore the dynamical features of a neural network model which presents two types of adaptative parameters : the classical weights between the units and the time constants associated with each artificial neuron. We examine the network power spectra (to draw some conclusions over the frequential behavior of the network) and we compute the stability regions to explore the stability of the model. We show that the network is sensitive to the variations of the mean values of the weights and the time constants (because of the temporal aspects of the learned tasks). Nevertheless, our results highlight the improvements in the network dynamics due to the introduction of adaptative time constants and indicate that dynamic recurrent neural networks can bring new powerful features in the field of neural computing.*

INTRODUCTION

Artificial neural network research essentially concerns feedforward networks which are now widely considered as powerful tools to approximate functions, to execute classification tasks or to act as an associative memory. Nevertheless, most of the feedforward neural networks have to perform tasks that we can consider as *static* : recognition of characters, patterns, images, sequences, etc. In contrast, recurrent neural networks have important capabilities not found in feedforward networks, including attractor dynamics and the ability to store information for latter use. Our purpose in this paper is to study dynamic recurrent neural networks which presents high-level temporal behaviour and can learn non-fixedpoint attractors. An algorithm that learns non-fixedpoint attractors and produces a desired temporal behaviour over a bounded interval was developed and is known as *Time-Dependent Recurrent Backpropagation (TDRBP)* (Pearlmutter 1989).

Dynamic recurrent neural networks are a variation of the traditional neural network model : they present two types of adaptative parameters : the classical weights between the units and the time constants associated with each artificial neuron (these time constants represent the membrane time constants of the biological neurons).

ARCHITECTURE OF THE NETWORK

We consider dynamic recurrent neural networks which are fully-connected (including self-connections) and governed by the following equations (Pearlmutter 1989) :

$$T_i \frac{dy_i}{dt} = -y_i + F(x_i) + I_i \qquad \text{where} \qquad x_i = \sum_j w_{ji}\, y_j \tag{1}$$

where y_i is the state or activation level of unit i and T_i is its time constant. $F(\alpha)$ is the squashing function (defined between -1.0 and 1.0).

[1]Research Assistant of the Belgian National Fund for Scientific Research - E-mail : jpd@pip.fpms.ac.be

Since we want our network to exhibit some particular temporal behaviour, the error function will be a functional defined as $E = \int_{t_0}^{t_1} q\,(\mathbf{y}(t), t)\, dt$ where t_0 and t_1 give the time interval during which the correction process occurs. The function $q\,(\mathbf{y}(t), t)$ is the cost function at time t which depends on the vector of the neuron activations $\mathbf{y}$ and on time.

After the introduction of new variables $p_i(t)$, we can derive the learning equations :

$$\frac{\partial E}{\partial w_{ij}} = \frac{1}{T_i} \int_{t_0}^{t_1} y_i\, F'(x_j)\, p_j\, dt \qquad \text{and} \qquad \frac{\partial E}{\partial T_i} = \frac{1}{T_i} \int_{t_0}^{t_1} p_i\, \frac{dy_i}{dt}\, dt \qquad (2)$$

These equations were first proposed by B. Pearlmutter in 1989 using ordered derivatives but can also be derived either using the calculus of variation, the Lagrange multiplier, or even from the theory of optimal control in dynamic programming using the Pontryaguin Maximum Principle (for more details see (Draye et al in press)).

INFLUENCE OF THE TIME CONSTANTS ON THE NETWORK DYNAMICS

Our goal is to study the influence of the supplementary adaptative parameter of the neural model : the time constants.

This study will be divided in two parts : first we will examine the network power spectra (to study the frequential behaviour of the network) and, secondly, we will compute the stability regions around fixedpoints (to qualitatively study the dynamical behaviour of the network with respect to the values of time constants).

Network power spectra

Our aim here is to characterize the network behaviour and its accessible dynamics by computing the power spectra of nodes. This characterization will give us interesting considerations concerning the influence of time constants on the frequential characteristics of the temporal sequences generated by dynamic recurrent neural networks.

We consider below 200 neuron networks whose weights and time constants are the realization of independent normal gaussian variables. For each network, 30 initial weights and time constants were chosen at random. The parameters mean value of the weights $\bar{w}$ and mean value of time constants $\bar{T}$ were varied and each resultant system was run with 10 random initial states of the neuron activations. No learning phase took place. The power spectrum of each node was computed using a 512-point fast Fourier transform ($T_{FFT} = 512$); the maximum detectable period equals 256.

The FFT data was summarized into two numbers per node : μ_i^P, the mean power over frequency and S_i, the entropy measure of power over frequency.

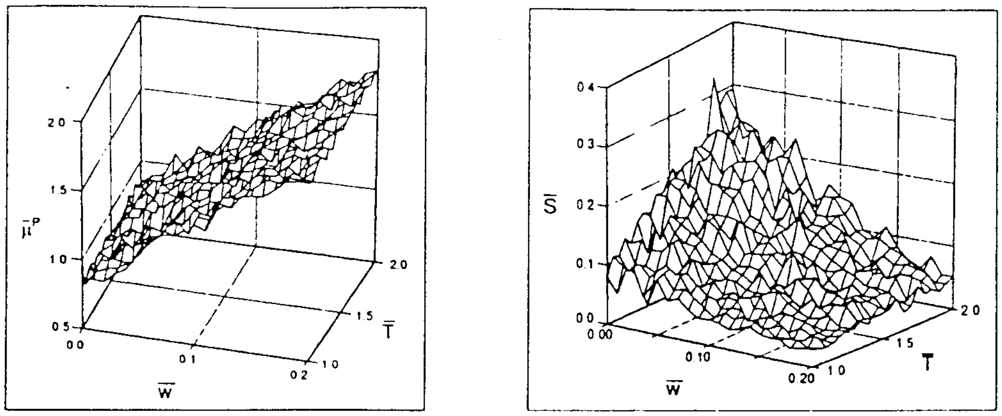
$$\mu_i^P = \frac{2}{T_{FFT}} \sum_{\omega} P_i(\omega) \qquad \text{and} \qquad S_i = -\sum_{\omega} P_i'(\omega)\, \ln P_i'(\omega) \qquad (3)$$

where $P_i(\omega)$ is the power spectrum at frequency ω at node i (P_i' denote a normalized power spectrum).

We averaged these values for all the nodes of the network and got two mean values : $\bar{\mu}^P$ and $\bar{S}$.

We can interpret the mean power measure $\bar{\mu}^P$ as an indication of the ranges of oscillations present in the network. The entropy measure $\bar{S}$ gives an indication of the width of the power spectrum : a single peak spectrum will have $\bar{S}$ equals 0. A complex periodic oscillation behaviour will have larger values of $\bar{S}$, large values indicate a chaotic oscillation behaviour with a broadband power spectrum (Renals 1990).

We simulated the network as described previously for each value of the $(\bar{w}, \bar{T})$ parameter pair. The weights mean value $\bar{w}$ varied from 0.0 to 0.2 whereas the time constants mean value $\bar{T}$ ranged from 1.0 to 2.0. The results are depicted on Figure 1 (mean power measure $\bar{\mu}^P$ and mean entropy measure $\bar{S}$).



(a) (b)

Figure 1: (a) Mean power measure $\bar{\mu}^P$ of a 200 neuron network versus weights mean value $\bar{w}$ and time constant mean value $\bar{T}$.(b) Mean entropy measure $\bar{S}$ of a 200 neuron network versus $\bar{w}$ and $\bar{T}$.

We see from Figure 1.a that an increase of $\bar{w}$ will increase the mean power $\bar{\mu}^P$. The proportional relation between $\bar{w}$ and $\bar{\mu}^P$ is due to the fact that a higher mean value of the weights makes the network evolve quickly to the saturation zones of the sigmoid. Figure 1.a also shows that an increase of the mean value of the time constants $\bar{T}$ has no effect on the frequential mean power $\bar{\mu}^P$ but it will affect the entropy measure.

Figure 1.b shows the entropy measure $\bar{S}$ versus $\bar{w}$ and $\bar{T}$. $\bar{S}$ is much higher for low values of $\bar{w}$: a quite normal fact because, for these values of $\bar{w}$, the network remains far away from the clipping zones and works in the linear part of the sigmoid function allowing wide oscillations. For a given $\bar{w}$, we see that the entropy measure increases with $\bar{T}$. Indeed, if the time constants T_i's have higher values, the network will settle itself more slowly to a stable state, allowing a more complex frequential behaviour.

In conclusion, $\bar{w}$ will affect the frequential mean power $\bar{\mu}^P$ and the entropy measure $\bar{S}$ because it determines the working region of the sigmoid where the network will mainly operate (linear, non-linear or saturation). When saturation occurs, the mean power measure will be nearly maximum whereas the oscillations (and thus the entropy measure) quite weak. The time constant mean value $\bar{T}$ will only modify the frequential behaviour and thus $\bar{S}$.

Computation of the stability regions

The analysis of the network power spectra gives us insights of the influence of time constants on the frequential behaviour of the network. An alternative study to characterize the influence of T_i's on the network dynamics is to compute the stability regions corresponding to some particular equilibrium points and to study their evolutions with respect to time constants.

Indeed, the success of regenerating a desired temporal behaviour, leading to a particular final state, from partial information is directly related to the stability boundaries of attraction of the corresponding final state. This success, as we will see, depends on the time constant values.

We will use the method described by Michel *et al* in 1982 that calculates the boundaries of stability regions using the classical Lyapunov functions. Moreover, we will improve the computation using an iterative method of refinement described in (Chiang 1989).

We are interested here in the stability region of a locally stable equilibrium point $\mathbf{y}^*$ of a dynamic recurrent neural network. The concept of stability regions is defined as the set of all initial conditions which have the property that any solution trajectories starting from them eventually approach the corresponding stable equilibrium point (Michel et al 1982). If we consider the particular case of our network, which can be described as a nonlinear dynamical system by the equation :

$$\frac{d\mathbf{y}(t)}{dt} = \mathcal{F}_{\text{net}}(\mathbf{y}(t)) \tag{4}$$

where $\mathcal{F}_{\text{net}}$ summarizes the action of the network (through eq. (1)) during the interval of processing. The solution curve of eq. (4) starting from $\mathbf{y}_0$ at time $t = t_0$ is called a trajectory denoted by $\phi(\mathbf{y}_0, t)$ (and thus $\phi(\mathbf{y}_0, 0) = \mathbf{y}_0$). The stability region of the equilibrium point can be written as :$\mathcal{S}(\mathbf{y}^*) = \{\mathbf{y}_0 \in \mathbb{R}^n;\ \lim_{t\to\infty} \phi(\mathbf{y}_0, t) = \mathbf{y}^*\}$. That means that $\mathcal{S}(\mathbf{y}^*)$ contains all the points of the state space $\mathbb{R}^n$ that will induce a curve that ends at $\mathbf{y}^*$.

The problem is now to estimate this domain of attraction, denoted by $\mathcal{S}$. A common approach is to use the critical level values of the Lyapunov functions associated to the system (Michel et al 1982). If $\mathcal{V}(\mathbf{y})$ is a local Lyapunov function of an equilibrium point $\mathbf{y}^*$, we call $\mathcal{W}$ an open set over which $\frac{d\mathcal{V}}{dt}(\mathbf{y})$ is strictly negative except at the equilibrium point $\mathbf{y}^*$ where it vanishes. We will next consider the following set $\mathcal{D}_{\mathcal{V}}(r) = \{\mathbf{y} \in \mathbb{R}^n;\ \mathcal{V}(\mathbf{y}) < r\}$ (with $r \in \mathbb{R}$). If $\mathcal{D}_{\mathcal{V}}(r)$ is contained in $\mathcal{W}$, then $\mathcal{D}_{\mathcal{V}}(r)$ is a subset of the complete stability region $\mathcal{S}(\mathbf{y}^*)$. The best estimate of the stability region of the equilibrium point $\mathbf{y}^*$ for the chosen Lyapunov function $\mathcal{V}(\mathbf{y})$ will be $\mathcal{D}_{\mathcal{V}}(r^*)$ where r^* is the largest constant such that $\mathcal{D}_{\mathcal{V}}(r^*) \in \mathcal{W}$.

Chiang *et al* in 1989 proposed an iterative method to improve the evaluation of the stability region. Once the best r^* value is obtained using the classical computation method. Chiang proposed a modification of the Lyapunov function to obtain a better estimate : $\mathcal{V}(\mathbf{y})$ is replaced by $\mathcal{V}_1\,[\mathbf{y} + \alpha.\mathcal{F}_{\text{net}}(\mathbf{y})]$ where α is a positive constant. It has been proved that the set $\mathcal{D}_{\mathcal{V}_1}(r^*)$ is also included in the exact stability region $\mathcal{S}(\mathbf{y}^*)$. The expansion scheme continues with the generation of new Lyapunov functions $\mathcal{V}_{k+1}(\mathbf{y}) = \mathcal{V}_k\,[\mathbf{y} + \alpha.\mathcal{F}_{\text{net}}(\mathbf{y})]$. At each step, a new stability region estimate is obtained using the same critical level value of the original Lyapunov function $\mathcal{V}(\mathbf{y})$: r^*.

In order to get exploitable results, we applied this method to a network of three neurons; the network was limited to such a small size just to allow us to visualize the stability regions in the phase-space $\mathbb{R}^3$ of the activations (y_1, y_2, y_3).

We trained the network to follow a trajectory starting at an initial state (y_1^0, y_2^0, y_3^0) of the three neurons and to evolve alone to the desired fixedpoint (y_1^*, y_2^*, y_3^*) corresponding to the initial state.

The first training set is analoguous to the truth table of a logical AND. Figure 2a depicts the four trajectories learned by the network : one can see that if we set the initial state of the network to (-1.0;-1.0;0.0), (-1.0;1.0;0.0), (1.0;-1.0;0.0), the system evolves towards the fixedpoint (-1.0;-1.0;-1.0) (denoted *a* on Fig. 2.a). Similarly, the initial state (1.0;1.0;0.0) leads the network to the fixedpoint (1.0;1.0;1.0) (denoted *b*). The activation of neuron 3 is always set to 0.0 at the beginning and it gives the result of the AND operation.

After the training, we modified the time constants in a decreasing way to examine the effect of the time constant values on the network dynamics (being careful that the network still exhibits the same fixedpoints behaviour). Figure 2.b shows the stabitity regions associated to the fixedpoints *a* and *b* and presents the comparison between the stability regions of the system with normal and small time constants. Inner regions are always associated with small T_i's. The axis labels are the respective activations y_i of the three neurons. The stability regions were computed with the method described previously. It is clear that the first effect of small time constants is to make the stability regions shrink dramatically.

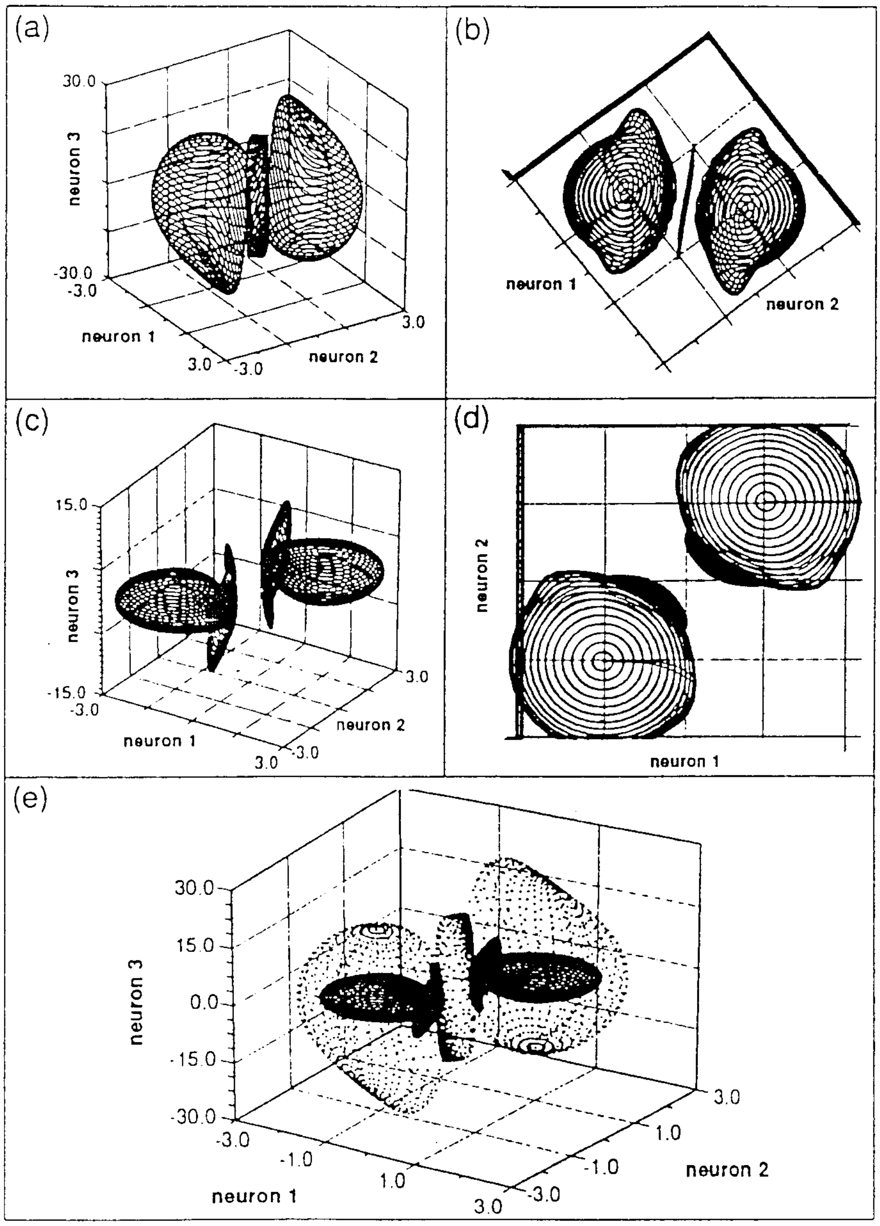


Figure 2: (a) shows the trajectories corresponding to a logical AND. (b) notes the comparison of the stability regions associated to fixedpoints a and b. (c) and (d) plot the dynamics with three fixedpoints. The axis labels are the respective activations y_i of the three neurons.

Afterwards, we computed the stability regions of these three fixedpoints (see Fig 2.c). If we decrease the time constants, the stability regions also shrinked. Nevertheless, if we go on decreasing the T_i's, we remark that the fixedpoint $(0.0; 0.0; 0.0)$ disappears and gives birth to two distinct different fixedpoints : approximately at $(0.3; 0.3; 0.0)$ and $(-0.3; -0.3; 0.0)$ (see Fig 2.d).

To sum up the effects of time constants, we can say that T_i's play an important role in the networ dynamics. They can heavily increase the dynamics and improve the features of the network but the variations can also induce deep modifications in the dynamical asymptotic behaviour.

Note that we did not simulate the classical exclusive-or problem because it requires a four neuro network and it would have been impossible to visualize the stability regions with 3D plots.

CONCLUSION

We have presented applied results from the analysis of complex dynamical systems to qualitatively and quantitatively study the dynamical behaviour of dynamic recurrent neural networks. We have studied the effects of the time constants T_i on the network. Our experiments demonstrate that $\bar{w}$ affects the frequential power (the amplitude of oscillations present in the network) and the entropy measure (proportionnal to the width of power spectrum). Indeed, high $\bar{w}$ values drive the network to the saturation zone of the sigmoid functions. The time constants mean value only affects the entropy measures allowing the network a more complex frequential behaviour. We reinforced the study of the influence of time constants by using methods for estimating the domain of attraction of an asymptotically stable equilibrium. We established the variation of the volume and the numbers of these regions with a variation of T_i's.

Our goal is to provide a strong theoretical basis for modeling and simulating dynamic recurrent neural networks. Therefore, the analysis of the dynamical behaviour is a necessary step to understand the capabilities and limitations of these powerful architectures that can deal with temporal sequences and signals.

Acknowledgements

This work is supported by the Belgian National Fund for Scientific Research (F.N.R.S.), by the Research Fund of the University of Brussels (U.L.B.) and by the EC through the ESPRIT program NeuroCOLT (Working Group Nr 8556).

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